*2006 IEEE International Conference on
Systems, Man, and Cybernetics
October 8-11, 2006, Taipei, Taiwan*

Design of Refined Grey Prediction Controller

Chia-Chang Tong and Enrique Barbieri, *Member, IEEE*

***Abstract*—The goal of this article is to offer an automatic PID parameter adjustment strategy by using a Grey Predictor model. The Grey Predictor model together with a first-order low-pass α filter generates effective estimated values of the plant output. The output error is then embedded in a control algorithm that automatically tunes the PID parameters. The resulting PID/Grey controller is shown to outperform the PID controller that is tuned via the standard quarter-decay ratio method. The paper gives a detailed description of the system structure, the design algorithm, and its implementation. Regulation and tracking performance of the proposed PID/Grey Predictor Controller are illustrated by means of computer simulation tests in the Matlab environment. Finally, a temperature regulation example is built by using LabVIEW programming and tested to validate the performance of the proposed controller scheme.**

***Index Terms*—Grey Predictor Controllers, Low-pass α Filter, LabVIEW, PID controller.**

I. INTRODUCTION

THE design of high quality controllers usually takes the following features into consideration: (a) simplicity in design and construction, (b) ease of operation, (c) stability margins, (d) low parameter-drifting, (e) fast system response, (f) minimal system overshoot, and (g) no output steady state error. The proportional-integral-derivative controller (PID) is frequently employed in industry because of these qualities. First proposed by Ziegler-Nichols in the 40s, the PID is widely adopted nowadays, although it has been under constant modification and improvement [1]-[5]. With the merits mentioned above, the PID controller is able to reduce response error via proportional control, and thus the process can track the input and respond properly; it can lower the output overshoot and reduce the response time through derivative control; and it can eliminate steady-state offset through integral control. Nevertheless, when large load variations and non-linear industrial control processes are involved, a set of fixed PID parameters cannot always obtain satisfactory control performance. Moreover, the fact that three parameters need to be adjusted usually requires that an experienced engineer be involved in the practical implementation of the PID controller. Since it is never easy to find a set of proper parametrical values, this compromises the high adaptability and convenience of the PID controller. It is always the case that improperly adjusted parameters cause unsatisfactory system response.

In this paper, a new structure is proposed to automatically adjust the parameters of a PID controller with the aid of a Grey-Predictor model that achieves better system response without any prior knowledge of the plant and without human operation.

II. SYSTEM OVERVIEW

Fig. 1 shows the block diagram of the proposed system structure of the PID/Grey predictor controller. It displays a closed-loop system with an adjustable PID controller whose regulating mechanism is driven by the Grey predictor.

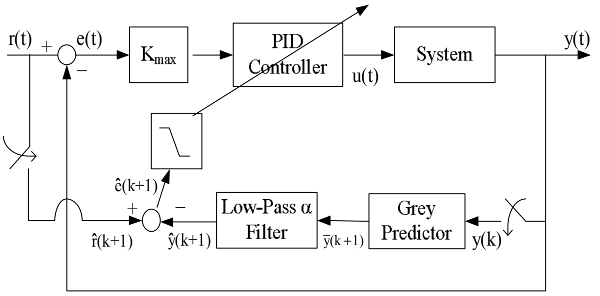

Fig. 1. System structure of Grey Predictor Controller

It should be noted that the three parameters are automatically regulated by the Grey predictor. Because the process of accumulated generating operation in the Grey predictor makes the prediction value slightly higher, a general first-order digital low-pass α filter is added to improve the prediction accuracy. The system output $y(t)$ is sampled into $y(k)$ and used by the Grey Predictor to generate the estimated output response $\bar{y}(k+1)$. The refined estimation $\hat{y}(k+1)$ is calculated by the low-pass α filter and the output prediction error value $\hat{e}(k+1)$ can be obtained by comparing with the sampled reference input $\hat{r}(k+1)$ (assumes slowly time-varying reference inputs). The Grey predictor generates a reliable prediction of the response error, which is used to automatically regulate the PID controller parameters. When compared to the standard PID, the computer simulated examples demonstrate that our proposed controller achieves a shorter rise time T_r, comparable settling time T_s, better overshoot OS suppression, comparable steady-state performance, and better response to transitions of a triangular reference input. A hardware implementation of a temperature controller is tested via LabVIEW virtual instrumentation to confirm the results of the computer simulation.

A. Grey Prediction Theory

Grey system theory is a useful tool for prediction. Its merit lies in the ability to make full use of limited data information

Chia-Chang Tong is with the Department of Electrical Engineering, Chienkuo Technology University, Changhua, Taiwan (e-mail: willtong@ctu.edu.tw).

Enrique Barbieri is with the Department of Engineering Technology, University of Houston, Houston, TX 77204-4021 (e-mail: ebarbieri@uh.edu).

1-4244-0100-3/06/$20.00 ©2006 IEEE

to predict future values. The grey prediction model conducts a so called "accumulated generating operation" on the original sequence. The resultant new series is used to establish a difference equation whose coefficients are found via the least-squares method. The accumulated generating series prediction model value is then obtained. The estimated prediction value in the time-domain is calculated by means of an inverse accumulated generating operation. Only a few original sequence elements are needed, and one does not have to assume the distribution of the sequence. The generated value is then used to establish a set of Grey difference equation and Grey pseudo differential equation. The model is called the Grey Model [6]-[12]. Generally, there are a few types used in the literature:

1) GM(1,1): This represents first-order derivative, containing one input variable, generally used for prediction purposes.
2) GM(1,N): This represents first-order derivative, but containing N input variables, for multi-variable analysis.
3) GM(0,N): This represents zero-order derivative, containing N input variables, for prediction purposes.

In this article, the GM(1,1) model is adopted to perform the prediction of system output response. The standard procedure is as follows:

Step 1: Collecting the original data sequence;

$$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), ..., x^{(0)}(n)\} \tag{1}$$

Step 2: Conducting an accumulated generation operation, AGO, on the original data sequence in order to diminish the effect of data uncertainty;

$$X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), ..., x^{(1)}(n)\} \tag{2}$$

Where $x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), k = 1,2,......,n$ (3)

Step 3: Establishing Grey difference equation and then calculating its background values;

$$x^{(0)}(k) = -az^{(1)}(k) + b \tag{4}$$

$$z_1^{(1)}(k) = 0.5\{x_1^{(1)}(k) + x_1^{(1)}(k-1)\} \tag{5}$$

Step 4: Constructing data matrix B and data vector Y;

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix} \tag{6}$$

$$Y_N = \left[X_1^{(0)}(2) \quad X_1^{(0)}(3) \quad \cdots \quad X_1^{(0)}(n)\right]^T \tag{7}$$

Step 5: Resolving the matrix;

$$Y_N = B\hat{a} \tag{8}$$

$$\hat{a} = \begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y_N \tag{9}$$

Step 6: Deriving the solution to the Grey difference equation;

$$x^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a}\right]e^{-ak} + \frac{b}{a} \tag{10}$$

Step 7: Conducting the inverse accumulated generation operation to obtain a prediction value

$$x^{(0)}(k+1) = (e^{-a} - 1)\left[x^{(0)}(1) - \frac{b}{a}\right]e^{-a(k-1)} \tag{11}$$

The output prediction value $y(k+1)$ as shown in Fig.1 is obtained by (11) through the GM(1,1) model.

B. First-order Digital Low-pass α Filter

A general first-order digital low-pass α filter following the GM(1,1) model is proposed to improve the prediction value. The filer is given by the following equation

$$\hat{y}(k+1) = (1-\alpha)\hat{y}(k) + \alpha y(k+1) \tag{12}$$

where α lies between 0 and 1. The structure of the first-order digital low-pass α filter is shown in Fig. 2. By plotting the estimation error vs. α as shown in Fig. 3 for a nominal second-order transfer function, it is found that α=0.6 minimizes the error.

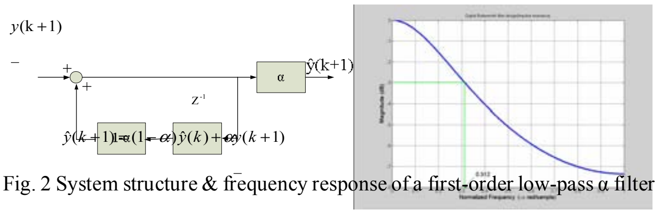

Fig. 2 System structure & frequency response of a first-order low-pass α filter

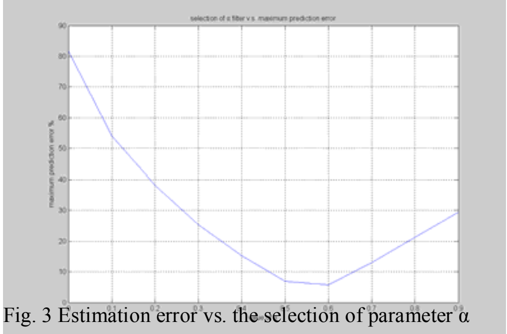

Fig. 3 Estimation error vs. the selection of parameter α

In order to explore the limitation of this prediction method, the unit feedback closed-loop structure with general second-order transfer function $T(s) = \frac{\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$ is used, where ζ is the damping ratio and ω_n is the natural frequency of the controlled process. Unit step responses are simulated and compared with estimations from Grey predictor and Grey predictor plus low-pass α filter. The sampling time is presumed to be 0.01 second. From Fig.2, the cutoff frequency of low-pass α filter (α=0.6) is 0.312π. In this case, the cutoff frequency is 15 Hz. In Fig. 4 and Fig. 5, the estimation error vs. natural frequency and damping ratio are compared. It is so obvious that the error is diminished by the low-pass α filter. For natural frequencies under 25 Hz, the estimation error of Grey predictor plus α filter is improved at least by 50 % than without the α filter. Due to the nature of the low pass filter, the error progressively gets worse for natural frequencies higher than 25 Hz.

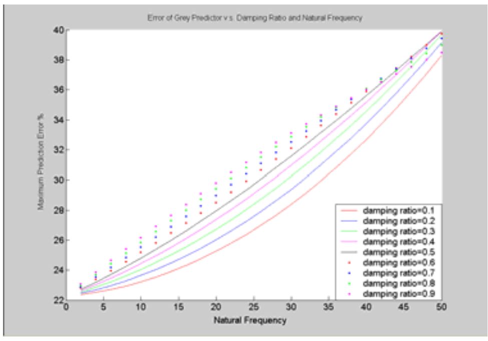

Fig. 4 Grey estimation error vs. natural frequency & damping ratio

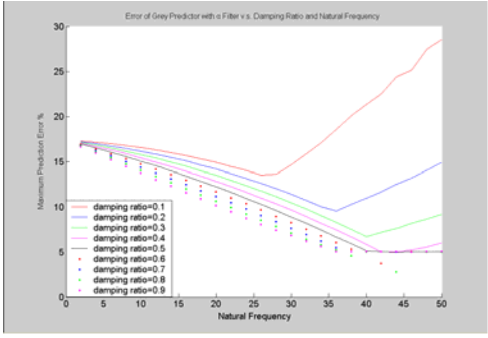

Fig. 5 Estimation error improved by α filter

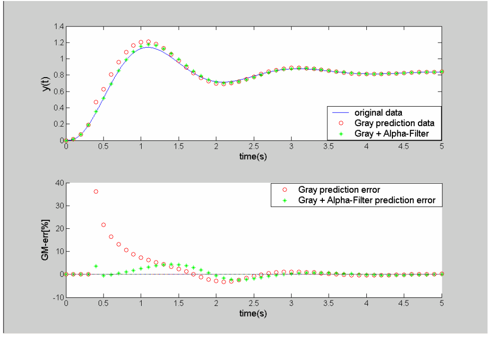

Fig. 6 Estimation error of unit step response

C. *PID Tuning Algorithm*

The system output prediction error ê(k+1) is utilized to adjust the three parameters of the PID controller. The function m(k) is a linear function of prediction error ê(k+1) with slope of 1/4, offset of 0.5, and limited in magnitude between 0 and 1. The constant $K_{\max}$ is the maximum gain the physical process can provide. The PID/Grey controller algorithm is then written as:

$$C(s) = K_{\max} \cdot m(k) \cdot (K_p + K_I \cdot \frac{1}{s} + K_D \cdot s) \tag{13}$$

where
$$m(k) = \begin{cases} 0, \mathit{If}\ 0.5 + 0.25 \cdot \hat{e}(k+1) \le 0 \\ 1, \mathit{If}\ 0.5 + 0.25 \cdot \hat{e}(k+1) \ge 1 \\ 0.5 + 0.25 \cdot \hat{e}(k+1), \mathit{else} \end{cases} \tag{14}$$

and

$$K_p = 1.5 - 0.5 \cdot m(k) \tag{15}$$

$$K_I = 1 \tag{16}$$

$$K_D = 1.25 + m(k) \tag{17}$$

III. MATLAB SIMULATION

In order to prove that Grey predictor controller can be extensively applied to regulate different control processes, we take a number of common control processes as examples, and simulate the process on the computer using Matlab software. The Grey predictor is used to control K_P, K_D, and K_I and enable over-all system response to shorten rising time and settling time, oppress maximum overshoot, and reduce steady state error to zero. It is found that Grey predictor controller can perform far much better than traditional PID controller. Not only do the rising time and settling time become distinctively shorter, but also the maximum overshoot is swiftly reduced and no steady state error occurs.

All of the following examples use the same proposed scheme, shown in Fig. 1, to form an auto-tuning PID controller via Grey-predictor. For the purpose of evaluation, the traditional PID controller is also simulated under the same condition. The parameter values of K_P, K_D, K_I are derived by quarter-decay ratio (QCR) method [5]. Matlab program simulation and mathematical argumentation are given to prove the feasibility of this novel auto-tuning regulator.

Example 1: The functional diagram of close-loop system as shown in Fig. 1, using unit step function as input and open-loop system as G(s) given in the following equation. Supposed the open-loop process is a third-order system with three real poles and no zero. The G(s) of this process is:

$$G(s) = \frac{1}{s^3 + 6s^2 + 11s + 6} \tag{18}$$

This is a type 0 system with constant steady state error for unit step input. In Fig. 7, the dotted line and real line are the unit step response curves derived by PID controller and Grey predictor controller respectively. As it shows in TABLE I, Grey predictor controller performs far much better than traditional PID controller in that not only does the rising time become distinctively shorter, but also the maximum overshoot is drastically reduced, and no steady state error occurs. However, the settling time is slightly higher.

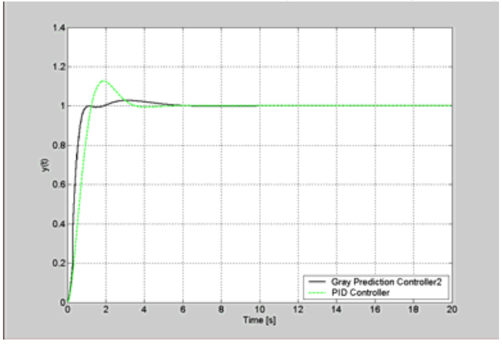

Fig. 7. Unit Step Response for PID and Grey Prediction Controller

TABLE I

Controller Type	Rising Time (Sec.)	Settling Time (Sec.)	Overshoot (%)
PID Controller	0.852	3.124	12.6085
Grey Controller	0.476	3.912	2.7675

K_p=15.2319, K_I=1.255, K_D=0.3137

To prove the superiority of Grey predictor controller over PID controller, we take the response overshoot of Grey predictor controller at 0.027675 and the settling time at 3.912 seconds in this example as the requirement to design a feasible PID controller, using root-locus method. To meet these requirements, the design procedure of PID controller is following.

At the beginning, we suppose that the maximum overshoot is 0.027675 and the settling time 3.912 seconds for designing a PD controller $k(s+z_c)$, the damping ratio will be 0.752289 and natural frequency be 1.41745. The characteristic values of close-loop will then lie on $-1.0663 \pm 0.9339j$. We put these values back to the characteristic equation of original system $s^3+6s^2+(11+k)s+(6+kz_c)=0$ and obtain k=-0.7432, z_c=-2.3819, but the gain k cannot be a negative. The result proves that no PD controller parameters can meet the design requirement. In other words, such a kind of PD controller does not exist. The function of integral control is added to eliminate steady state error after PD controller design achieves the required transient response. Thus, no PID controller can meet the requirement.

Example 2: The functional diagram of close-loop system as shown in Fig. 1, using unit step function as input and open-loop system as G(s) given in the following equation. Supposed the open-loop process is a fourth-order system with two pair of conjugate poles and one real zero. The G(s) of this process is:

$$G(s)=\frac{s+5}{s^4+16s^3+93s^2+152s+84} \tag{19}$$

This is a type 0 system with constant steady state error for unit step input. In Fig. 8, the dotted line and real line are the unit step response curves derived by PID controller and Grey predictor controller respectively. As it shows in TABLE II, Grey predictor controller performs far much better than traditional PID controller in that not only do the rising time and settling time become distinctively shorter, but also the maximum overshoot is drastically reduced, and no steady state error occurs.

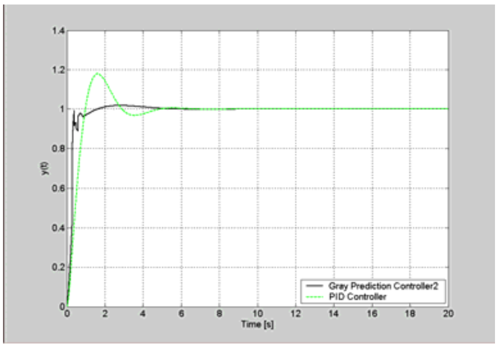

Fig. 8. Unit Step Response for PID and Grey Prediction Controller

TABLE II

Controller Type	Rising Time (Sec.)	Settling Time (Sec.)	Overshoot (%)
PID Controller	0.689	4.161	17.9344
Grey Controller	0.221	1.204	1.8072

K_p=53.5788, K_I=0.964, K_D=0.241

The same mathematical argumentation is used to prove the superiority of Grey predictor controller over PID controller. We take the response overshoot of Grey predictor controller at 0.018072 and the settling time at 1.204 seconds in this example as the requirement to design a feasible PID controller, using root-locus method. First, we design a PD controller $k(s+z_c)$ to meet the transient response requirements. The desired damping ratio will be 0.78744 and natural frequency be 2.65077. The characteristic values of close-loop will then lie on $-2.0873 \pm 1.6339j$. We put these values back to the characteristic equation of original system $s^4+16s^3+(93+k)s^2+(152+k+kz_c)s+(84+kz_c)=0$ and obtain k=-23.1857, z_c=-0.4445, but the gain k cannot be a negative. The result proves that no PD controller parameters can meet the design requirement. In other words, such a kind of PD controller does not exist. The function of integral control is added to eliminate steady state error after PD controller design achieves the required transient response. Thus, no PID controller can meet the requirement.

Example 3: The functional diagram of close-loop system as shown in Fig. 1, using unit ramp, unit square wave, and unit triangle functions as inputs and open-loop system as G(s) given in the following equation. Supposed the open-loop process is a third-order system with one real pole, one pair of conjugate poles, and two real zeros. The G(s) of this process is:

$$G(s)=\frac{s^2+6s+8}{s^3+10s^2+6s+12} \tag{20}$$

This is a type 0 system with constant steady state error for unit step input and infinite steady state error for unit ramp input. In Fig. 9, the dotted line and real line are the unit ramp response curves derived by PID controller and Grey predictor controller respectively. Fig. 10 and Fig.11 are the unit square wave response curves and the unit triangle wave response curves respectively. The parameter values of K_P, K_D, K_I are derived by quarter-decay ratio (QCR) method. In TABLE III, performing comparisons between PID and Grey Prediction Controller are made. For tracking evaluation, the Grey Prediction Controller outplays the traditional PID in every aspect.

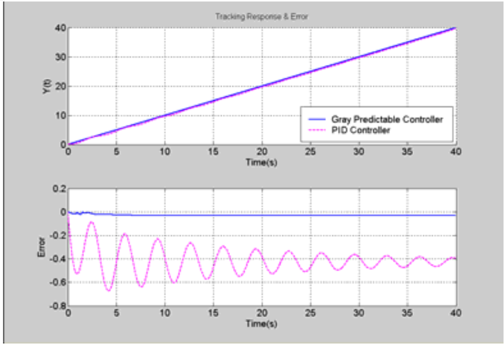

Fig. 9. Tracking Response for PID and Grey Prediction Controller

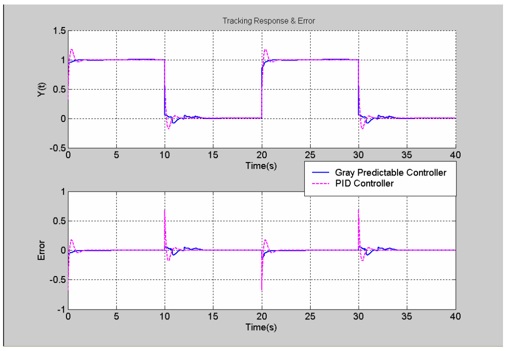

Fig. 10. Tracking Response for PID and Grey Prediction Controller

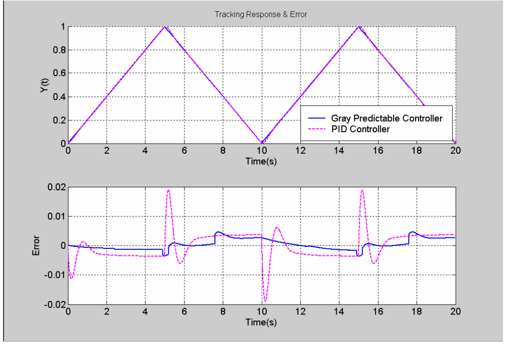

Fig. 11. Tracking Response for PID and Grey Prediction Controller

TABLE III

Controller Type	PID Controller	Grey Controller
Ramp: Steady State Error	0.42	0.03
Square Wave 0.05 Hz : Overshoot %	17.61	8.21
Triangle 0.1 Hz: Max. Tracking Error	0.0181	0.0046

$K_p=12.528$, $K_I=0.15$, $K_D=0.0375$

IV. HARDWARE IMPLEMENTATION BY LABVIEW

A temperature regulation experiment by using LabVIEW programming is under test [13]-[16]. The functional block diagram of this test is shown in Fig 12.

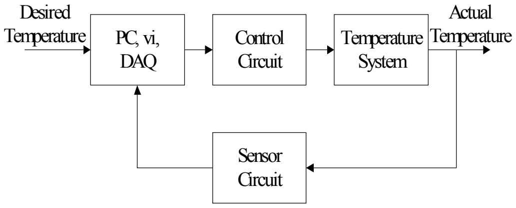

Fig. 12. Temperature Control Scheme

The sampling rate, 0.5 second, is selected because of the nature of temperature response. The frequency of PWM modulation is set to 200 Hz. The output response is recorded from staring point, room temperature, to the desired temperature, 60 °C. A ventilation fan is blowing to circulate the air around the temperature actuator in order to make some disturbance during the experiment. The virtual instrumentation front panel is shown in Fig. 13 and the output response is displayed in Fig. 14 for the Auto-tuning Grey-prediction controller. The same experiment conditions applied to a traditional PID controller. The parameter values of $K_P=555$, $K_D=0.75$, and $K_I=3$ are derived by quarter-decay ratio (QCR) method. The output response is displayed in Fig. 15 for the traditional PID controller.

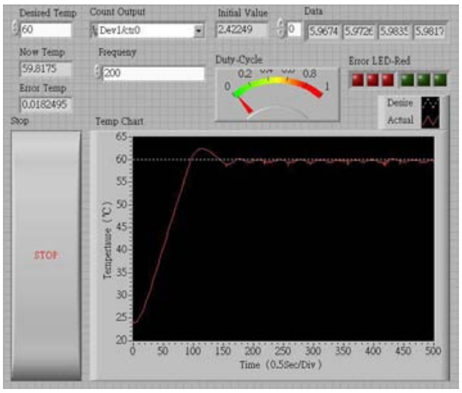
Desired Temp 60
Count Output Dev1/ctr0
Initial Value 2.42249
Data 0 5.9674 5.972 5.983 5.9817
Now Temp 59.8175
Frequeny 200
Error Temp 0.0182495

Fig. 13. Front Panel of Grey Prediction Controller

The essential specifications of both transient and steady-state response are calculated and listed in TABLE IV for comparison, including overshoot, rising time, 2% settling time and steady-state error. It is obvious that the proposed auto-tuning Grey-prediction controller is far better than the traditional PID controller from the graphs as well as the comparison table. In transient response, the proposed controller scheme is slightly faster than the traditional PID controller. The overshoot is effectively reduced from 10% to 6.6%. The settling time for the Grey-prediction controller is 81.5 second. There is no steady-state error. However, the response of the PID controller becomes a slow-decaying oscillation, ±2.7% at 250 second. It approaches zero eventually, but takes a lot of time. The auto-tuning Grey-prediction controller improves the transient and steady-state response in every aspect without doubt.

From the data collected in this test, we conclude that the Grey prediction controller expedites the process as well as effectively oppresses the overshoot and disturbance.

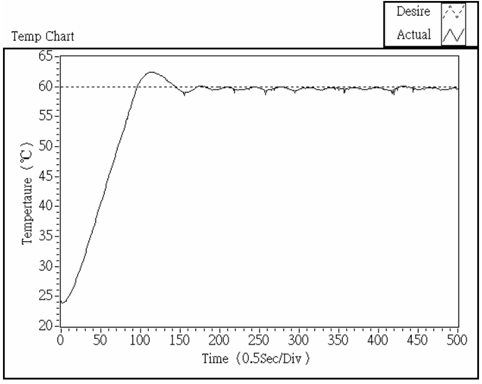

Fig.14. Output Response of the Auto-tuning Grey-Prediction Controller

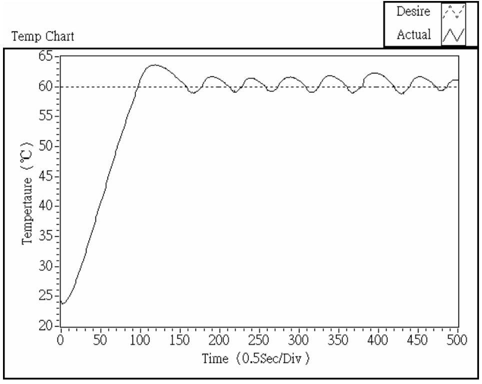

Fig. 15. Output Response of the Traditional PID Controller

TABLE IV: Transient and Steady-state Performance Comparison

	Overshoot (%)	Rising Time (sec.)	Settling Time (sec.)	Steady-state error
PID Controller	10.0	34.5	xxx	xxx
Grey Predictor Controller	6.6	33.5	81.5	0

Note: K_P=555, K_I=3,K_D=0.75

V. CONCLUSION

We investigate a feasible system structure of Grey prediction controller. The uniqueness of this design lies in the use of the output response prediction error values produced by Grey predictor to regulate PID controller parameters. Accordingly, it is able to deal with the possible variation of system responses at the very first stage. It can not only actively promote the responses efficiency of transient response, but also passively prevent disturbance. We conclude that there are four important contributions in this paper:

(1) It proves the regulation function of Grey predictor controllers in unit step response by means of Matlab program simulation and mathematical argumentation. In transient response, it will effectively fasten rising time, shorten settling time, and oppress overshoot; meanwhile, in steady state response, it is able to reduce steady state error to zero and achieve what traditional PID controller cannot perform.

(2) It verifies the tracking function of Grey predictor controllers in unit ramp response, square wave response, and triangle wave response by means of Matlab program simulation. The proposed system scheme can perform very well without any parameter adjustments. On the contrary, traditional PID controller does lousy jobs. A set of PID parameter derived by quarter-decay ratio (QCR) method does not adapt to the tracking task automatically.

(3) A hardware implement is built to run the test. By using the virtual instrument by LabVIEW programming, a Grey prediction controller is developed. The test results are promising and the feasibility is verified.

(4) It validates the flexibility of the proposed controller since we do not have any *prior* knowledge about the controlled process in the hardware implement.

VI. FURTHER RESEARCH

It is suggested that in the near future we can make an attempt to: (a) conduct practical tracking control experiment by using LabVIEW programming along with data-acquisition card in order to obtain empirical evidence; (b) use PSoC (programmable system-on-chip) devices from Cypress to miniaturize this controller for practical industrial applications; (c) further improve the present Grey prediction controller and enhance its performance with the aid of artificial intelligence; (d) develop other kinds of feasible system structure for controller with Grey system theory

ACKNOWLEDGMENT

The authors want to heartily thank the Chienkuo Technology University, for this article was partially supported by the project under CKIT-RP-EE-002-006-A.

REFERENCES

[1] M. R. Katebi, and M. H. Moradi, "Predictive PID Controllers," *IEE Proc-Control Theory Appl.*, Nov. 2001, vol. 148, no. 6 , pp.478-487.

[2] Hyun-Joon Cho, Kwang-Bo Cho, and Bo-Hyeun Wang, ''Automatic Rule Generation Using Genetic Algorithms for Fuzzy-PID Hybrid Control'', *IEEE Intemational Symposium on Intelligent Control*, pp. 271-276, September 1996.

[3] K. J. Astrom, "Toward Intelligent Control", *IEEE Control Systems Magazine*, pp. 60-64, April 1989.

[4] Rongfu Luo, S. Joe Qin, and Dapang Chen, "A New Approach to Closed Autotuning for PID Controllers," *Proceedings of the American Control Conference*, pp. 348-352, June 1998.

[5] C. C. Hang, K. J. Astrom, W. K. Ho, "Refinements of The Ziegle-Nichols Tuning Formula", *IEE Proceedings-D*, vol. 138, no. 2, March 1991, pp. 111-118.

[6] J. L. Deng, "Introduction to Grey System Theory," *The Journal of Grey System*, vol. 1, 1989, pp. 1-24.

[7] J. L. Deng, "Grey Differential Equation," *The Journal of Grey System*, Vol.1, 1993, pp. 1-14.

[8] Jet-Chau Wen, Kuo-Hsun Huang, Kun-Li Wen, "The Study of GM(1, 1) Model," *Journal of the Chinese Institute of Engineering*, 2000, vol. 23, no.5, pp. 583-589.

[9] Kun-Li Wen, Ting-Cheng Chang, Wei-Che Chang, Mei-Li You, "The Study of Missing Point in GM(1, 1) Model," *IEEE International Conference on System, Man and Cybernetics*, Oct. 2000, vol. 5, pp. 3384-3387.

[10] K. L. Wen, T. C. Chang, H. T. Chang, M. L. You, "The Adaptive α in GM(1, 1) Model," *IEEE International Conference on System, Man and Cybernetics*, Oct. 1999, pp. 304-308.

[11] M. L. You and K. L. Wen, "The Error Analysis in GM(1, 1) Model," *Journal of Chinese Grey System*, 2000, vol. 3, no.1, pp. 63-70.

[12] C. Y. Yang and J. J. Chou, "Entropy on Grey Relational Analysis," *The Journal of Grey System*, vol. 13,2001, pp. 313-320.

[13] Bishop, R. H., Learning with LabVIEW, Addison-Wesley, 1999.

[14] Wells, L. K. and Travis, J., LabVIEW for Everyone Graphical Programming Made Even Easier, Prentice Hall, Inc., Upper Saddle River, New Jersey, 1997.

[15] Essik, J., Advanced LabVIEW Labs, Prentice Hall, Inc., Upper Saddle River, New Jersey, 1999.

[16] Chugani, M. L., LabVIEW Signal Processing, Prentice Hall, Inc., Upper Saddle River, New Jersey, 1998.