AN N-STEP DEADBEAT REGULATOR

Enrique Barbieri
Department or Engineering Technology
University of Houston
Technology T2 304-D
Houston, Texas 77204-4021 USA
email: ebarbieri@uh.edu

ABSTRACT
The coefficients of a SISO compensator for constant set-point regulation in n^{th}-order LTI systems are synthesized using state-space methods without the need for observers so that deadbeat behavior is achieved. The compensator generates a control sequence that steers the error to zero in precisely n sampling steps. For second order systems the compensator can be implemented as a traditional *PI-Lead* or *PID* controller. For higher order systems, the second-order controller works as a starting point with additional tuning done if needed. Three computer-simulated examples are included to illustrate the results.

KEY WORDS: Deadbeat design

1 Introduction

Constant output set-point regulation remains as one of the most common problems in process control. A control strategy that forces the system to exhibit the so-called *deadbeat* behavior is particularly interesting and is the focus of this article. The deadbeat behavior in an n^{th}-order, linear, time-invariant, discrete-time system is achieved when the output is steered to the desired value and maintained there in a finite number of sampling instants N_s, that is, in N_sT_s seconds where T_s is the sampling time. A minimum-time response is obtained when the integer N_s is minimum.

In order to force the deadbeat behavior it is necessary to have all discrete-time poles of the closed-loop transfer function at the origin. The design of a pole-zero compensator for Single-Input Single Output (SISO) systems of order higher than two via conventional techniques such as Root-Locus may not be trivial once the constraint of having all closed-loop poles at zero is imposed. In SISO systems it is natural to take the transfer unction approach whereby, for minimum-phase systems and a given type of reference input, the pole-zero structure of the compensator is deduced by ensuring (1) that the error signal go to zero in the minimum number of steps; and (2) that the resulting compensator be physically realizable. A slight modification is introduced for plants with poles or zeros on or outside the unit circle [1]. In [2] deadbeat control designs are used for feed-forward control to reduce overshoots and settling times. A polynomial algebra approach is employed in [3] to derive a necessary and sufficient condition for the existence of a causal stabilizing deadbeat controller, for ripple free performance, independent of the initial state of the plant, and for each admissible input. A deadbeat-like controller is proposed in [6] for global asymptotic stability of output-saturated linear systems.

In [7], solving an l1-norm minimization problem a simple design method is presented for obtaining deadbeat controllers with given settling steps to guarantee the robust stability and to minimize the worst case steady-state controlled error. A suboptimal deadbeat controller design for a plant with measurement noise is proposed in [10] that uses a non-minimal order observer. It's been recently established that a deadbeat controller is relatively optimal [13].

Deadbeat controller design focusing on robustness issues are described in [8, 9]. Simultaneous time-domain and frequency-domain performance is improved in [11] by considering deadbeat tracking and $H-\infty$ methods. Multi-variable designs for deadbeat regulation requiring observers include [4, 5, 10]. An application of deadbeat control to a three link under-actuated manipulator that avoids excessive overshoots is described in [12]. In [14], a PID controller results in a maximally deadbeat design whereby it is determined for the given plant, the smallest circle within the unit circle wherein the closed loop system characteristic roots may be placed by PID control.

The necessary condition for deadbeat behavior can also be cast as a pole-placement problem. Therefore, it is not surprising that the literature is abundant on deadbeat regulator designs that are based on state-space techniques. The article [15] provides an abundant source of references on deadbeat regulator designs especially for multi-input systems. In essence, the gain in the full-state feedback controller

$$u(k) \ = \ Kx(k); \qquad K \in \Re^{q\times n}$$

applied to the discrete-time system

$$x(k+1) \ = \ \Phi x(k) + \Gamma u(k)$$

where $x \in \Re^n$, $u \in \Re^q$, is tuned so that the closed-loop system matrix $\Phi + \Gamma K$ is nilpotent, that is, $(\Phi+\Gamma K)^{N_s} = 0$ for some integer $N_s > 0$. In these studies it is assumed that the state x is available or that an observer is incorporated in the design.

Unlike previous works, in this article state-space ideas are utilized to synthesize directly the *coefficients* of a pole-zero compensator in the standard unity-feedback configuration for n^{th}-order SISO systems. The procedure does not need an observer because the state vector is not required. The system exhibits deadbeat behavior in $N_s = n$ steps. The design is not necessarily minimum-time but the response is still quite fast. Section 2 states the problem. The compensator is synthesized in Section 3 where the main result is presented and the special case of $n = 2$ is considered. The results are illustrated via numerical examples in Section 4 and conclusions are given in Section 5.

2 Problem Statement

We consider a linear, time-invariant, n^{th}-order differential equation of the form

$$\tilde{y}^n + \sum_{j=1}^{n} a_{n-j}\tilde{y}^{n-j} = \sum_{j=0}^{m} b_{m-j}u^{m-j} \tag{1}$$

where u is the control, $m \leq n$, the initial "rest" output conditions are imposed

$$\left[y(0-);\ \dot{y}(0-);\ \cdots;\ y^{n-1}(0-)\right] = [y_0;\ 0;\ \cdots;\ 0] \tag{2}$$

and it is required to steer (1) to the final "rest" output conditions

$$\left[y(t_f);\ \dot{y}(t_f);\ \cdots;\ y^{n-1}(t_f)\right]' = [r;\ 0;\ \cdots;\ 0] \tag{3}$$

where overdots and exponents of continuous-time signals denote time differentiation. The functions $\tilde{y}(t)$ and $y(t)$ are related by

$$y(t) = \tilde{y}(t - t_d)$$

where $t_d \geq 0$ represents a time delay inherent in the system.

We take the approach of synthesizing a controller (regulator) in the discrete-time domain. To that end, the plant transfer function $G_p(s) = G_0(s)e^{-t_d s}$

$$G_p(s) \ = \ \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}\ e^{-t_d s} \tag{4}$$

admits the discretized transfer function

$$G(z) \ = \ \frac{1 - z^{-1}}{z^N}\mathcal{Z}_\Delta\left[\frac{G_0(s)}{s}\right] \tag{5}$$

where the delay is $t_d = NT_s + (1 - \Delta)T_s$, the sampling time is T_s, $0 < \Delta \leq 1$, $N \geq 0$ is an integer, and $\mathcal{Z}_\Delta$ is the modified Z-Transform.

In this article, we consider first the delay-free case, that is, $t_d = 0$. Then, a minimal realization of (1) is

$$\begin{aligned} \dot{x}(t) &= Ax(t) + bu(t); \quad x(0) \quad \text{given} \\ \tilde{y}(t) &= y(t) \ = \ c'x(t) \ + \ du(t) \end{aligned} \tag{6}$$

and its *zoh*-discretization is

$$\begin{aligned} x(k+1) &= \Phi x(k) + \Gamma u(k); \quad x(0) \quad \text{given} \\ y(k) &= c'x(k) \ + \ du(k) \end{aligned} \tag{7}$$

where $x \in R^n$, $u \in R^1$, $y \in R^1$ are the state, control and output vectors, respectively, and the matrices A, B, c, d,

$$\phi = e^{AT_s}; \quad \text{and } \Gamma = \left\{\int_0^{T_s} e^{A\tau}d\tau\right\}b$$

are appropriately dimensioned. Throughout the paper, we let $(\cdot)'$ denote the transpose of the indicated vector or matrix. Minimality is equivalent to {A,B} controllable, and $\{A, c\}$ observable. Also, except for some peculiar values of T_s, the pair $\{\Phi, \Gamma\}$ is reachable and the pair $\{\Phi, c\}$ is observable.

We are interested in developing the discrete-time filter $G_d(z)$ which operates on the scalar error sequence, $e(k) = r - y(k)$, and produces the scalar control sequence $u(k)$ that forces $e(n) = 0$. We refer to $G_d(z)$ as an n-step deadbeat compensator.

3 Synthesis of $G_d(z)$

Let λ be an eigenvalue of A, $\mu = e^{\lambda T_s}$ an eigenvalue of Φ, and M the modal matrix of Φ, that is, $M^{-1}\Phi M = J$ where J is the Jordan form of Φ. We further assume in this article that $\lambda = 0$ may have multiplicity of only one.

Lemma 1 *Let $\mathcal{N}(\cdot)$ and $\mathcal{R}(\cdot)$ denote the Null and Range spaces, respectively, of the indicated matrices. Then,*

- $x \in \mathcal{N}\{I - \Phi\} \quad \Longrightarrow \quad M^{-1}x \in \mathcal{N}\{I - e^{JT_s}\}$

and, if $\lambda \neq 0$ ($\mu \neq 1$), then

- $x \in \mathcal{R}\{(I - \Phi)^{-1}\Gamma\} \quad \Longrightarrow \quad x \in \mathcal{R}\{-A^{-1}b\}$

Proof: *The first item follows from*

$$I - \Phi \ = \ I - e^{AT_s} \ = \ I - Me^{JT_s}M^{-1} \ = \ M(I - e^{JT_s})M^{-1}$$

and the second item follows because

$$\Gamma = M\left\{\int_0^{T_s} e^{J\tau}d\tau\right\}M^{-1}b = -M(I - e^{JT_s})J^{-1}M^{-1}b$$

and therefore

$$\begin{aligned} (I - \Phi)^{-1}\Gamma &= -M(I - e^{JT_s})^{-1}M^{-1}M(I - e^{JT_s})J^{-1}M^{-1}b \\ &= -A^{-1}b \end{aligned}$$

The initial output conditions (2) and the corresponding equilibrium or steady-state value of the state vector x_{ss} are held by the constant control u_{ss}. Then, the system of equations

$$x_{ss} \ = \ \Phi x_{ss} \ + \ \Gamma u_{ss} \tag{8}$$

$$y_0 \ = \ c'x_{ss} \ + \ du_{ss} \tag{9}$$

is satisfied and may admit the solution $u_{ss} = 0$ in the following instances:

- $\lambda \neq 0$ ($\mu \neq 1$): since $u_{ss} = 0$, then from *Lemma 1*, the nullspace of $(I - \Phi)$ contains only the origin ($x_{ss} = 0$) forcing $y_0 = 0$. This is too restrictive and will not be considered any further.

- $\lambda = 0$ ($\mu = 1$): in this case, $u_{ss} = 0$ in Equation (8) implies that x_{ss} is an eigenvector of Φ corresponding to the eigenvalue $\mu = 1$. Note that observability of the pair $\{\Phi, c\}$ prevents x_{ss} from being orthogonal to c. Therefore, $u_{ss} = 0$ in Equation (9) implies $y_0 = c' x_{ss}$. This is basically a reflection of the fact that if $\lambda = 0$ ($\mu = 1$), then in conventional terminology, the process is Type I, and is thus capable of tracking a constant reference with zero steady-state error. In this case the controller does not need to introduce integral action.

Denote by $\mathcal{C}_{j-1}$ the $n \times j$ matrix

$$\mathcal{C}_{j-1} = \left[\Phi^{j-1}\Gamma \vdots \Phi^{j-2}\Gamma \cdots \Phi\Gamma \vdots \Gamma\right]$$

known as the reachability matrix associated with (7) when $j = n$, and denote by $\mathcal{O}$ the $n \times n$ observability matrix

$$\mathcal{O} = \left[c \vdots A'c \vdots (A^2)'c \cdots (A^{n-1})'c\right]'$$

associated with (6). The main result of this article is stated as follows:

Theorem 1 *The output of system (1) is steered from (2) to (3) in exactly nT_s seconds (n steps) and kept there by the control sequence $\{\bar{u}(k) + u_{ss},\ k = 0, 1, \cdots, n-1;\ \bar{u}_n + u_{ss},\ k \geq n\}$ where u_{ss} is found from Equations (8) and (9), and $\bar{u}(k)$ is generated by the filter*

$$G_d(z) = \frac{\bar{U}(z)}{E(z)} = \frac{\sum_{k=0}^{n-1} v(k) z^{n-k-1} + \frac{1}{z-1} v_n}{\sum_{k=0}^{n-1} \eta(k) z^{n-k-1}} \quad (10)$$

where the filter coefficients are computed as follows:

$$V_{n-1} = \begin{bmatrix} v(0) \\ v(1) \\ \vdots \\ v(n-1) \end{bmatrix}$$

$$= \mathcal{C}_{n-1}^{-1} \mathcal{O}^{-1} \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} - v_n \begin{bmatrix} d \\ c'b \\ c'Ab \\ \vdots \\ c'A^{n-2}b \end{bmatrix} \right\}, \quad (11)$$

$$\begin{aligned} \eta(0) &= 1 - dv(0), \\ \eta(j) &= 1 - c'\mathcal{C}_{j-1} V_{j-1} - dv(j); \quad j = 1, \cdots, n-1 \end{aligned}$$

and

$$v_n = \begin{cases} 0 & \lambda = 0 \textit{ simple eigenvalue of } A \\ \frac{1}{d - c'A^{-1}b} & \lambda = 0 \textit{ not an eigenvalue of } A \end{cases}$$

Proof: *The stated regulation problem is equivalent to regulating from* the origin, *that is, from* $\{\bar{x}(0) = 0;\ \bar{y}(0) = 0\}$ *to* $\{\bar{x}(n);\ \bar{y}(n)\}$ *as can be easily seen by the change of variables*

$$\bar{u} = u - u_{ss}; \quad \bar{x} = x - x_{ss}; \quad \bar{y} = y - y_0$$

leading to

$$\begin{aligned} \bar{x}(k+1) &= \Phi\bar{x}(k) + \Gamma\bar{u}(k); \quad \bar{x}(0) = 0 \\ \bar{y}(k) &= c'\bar{x}(k) + d\bar{u}(k) \end{aligned} \quad (12)$$

Since $\bar{u} = constant$ for $t \geq t_f$, the set of equations

$$\begin{aligned} \bar{y} &= c'\bar{x} + d\bar{u} \\ \dot{\bar{y}} &= c'A\bar{x} + c'b\bar{u} + d\dot{\bar{u}} \\ \vdots & \quad \vdots \\ (\bar{y})^{n-1} &= c'A^{n-1}\bar{x} + \cdots + d(\bar{u})^{n-1} \end{aligned}$$

yields at the final time $t_f = nT_s$, with $\bar{Y}(nT_s) = [\bar{r} = r - y_0;\ 0; \cdots; 0]$, the following relation

$$\begin{aligned} \bar{Y}(nT_s) &= \mathcal{O}\bar{x}(nT_s) + \begin{bmatrix} d \\ c'b \\ c'Ab \\ \vdots \\ c'A^{n-2}b \end{bmatrix} \bar{u}(nT_s) \\ &= \mathcal{O}\bar{x}(nT_s) + h\bar{u}(nT_s) \end{aligned} \quad (13)$$

On the other hand, using (12), one obtains

$$\bar{x}(n) = \Phi^n \bar{x}(0) + \mathcal{C}_{n-1}\bar{U} \quad (14)$$

where

$$\bar{U} = [\bar{u}(0);\ \bar{u}(1);\ \cdots;\ \bar{u}(n-1)]'$$

Combining Equations (13) and (14), and using $\bar{x}(0) = 0$, we solve for $\bar{U}$:

$$\bar{U} = \mathcal{C}_{n-1}^{-1} \mathcal{O}^{-1} \left\{ \begin{bmatrix} \bar{r} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} - \bar{u}_n \begin{bmatrix} d \\ c'b \\ c'Ab \\ \vdots \\ c'A^{n-2}b \end{bmatrix} \right\}. \quad (15)$$

Next, $\bar{u}_n = 0$ if the spectrum of A contains $\lambda = 0$, so that in effect the integral action is eliminated from $G_d(z)$; otherwise, from item 2 of Lemma 1 *and Equation (8) rewritten in terms of the "bar" variables, it follows that*

$$\bar{x}_{ss} = -\bar{u}_{ss} A^{-1} b = -\bar{u}_n A^{-1} b$$

which, together with Equation (13), yields

$$\bar{u}_n = \frac{\bar{r}}{d - c'A^{-1}b} \triangleq v_n \bar{r}.$$

Substituting into Equation (15), we obtain

$$\bar{U} = V_{n-1}\bar{r}$$

where V_{n-1} was defined in (11).

The error sequence $\bar{e}(k) = \bar{r} - \bar{y}(k) = e(k)$ is found to be

$$\begin{aligned}
\bar{e}(0) &= \bar{r} - c'\bar{x}(0) - d\bar{u}(0) = [1 - dv(0)]\,\bar{r} \\
\bar{e}(1) &= \bar{r} - c'\bar{x}(1) - d\bar{u}(1) = [1 - c'\Gamma v(0) - dv(1)]\,\bar{r} \\
&\vdots \quad \vdots \\
\bar{e}(j) &= [1 - c'\mathcal{C}_{j-1}V_{j-1} - dv(j)]\,\bar{r} \stackrel{\triangle}{=} \eta(j)\bar{r}
\end{aligned}$$

Now, since $\bar{u}(k)$ and $\bar{e}(k)$ can be written as

$$\bar{u}(k) = \bar{u}(0)\delta(k) + \bar{u}(1)\delta(k-1) + \cdots +$$

$$+\bar{u}(n-1)\delta(k-n+1) + \sum_{j=0}^{\infty} \bar{u}_n \delta(k-n-j)$$

and

$$\bar{e}(k) = \bar{e}(0)\delta(k) + \bar{e}(1)\delta(k-1) + \cdots + \bar{e}(n-1)\delta(k-n+1),$$

their $\mathcal{Z}$-Transforms are

$$\bar{U}(z) = \left[\frac{v(0)z^{n-1} + v(1)z^{n-2} + \cdots + v(n-1)}{z^{n-1}}\right]\bar{r} +$$

$$+\frac{1}{z^{n-1}}\frac{1}{z-1}v(n)\bar{r}$$

and $\bar{E}(z) = E(z)$

$$E(z) = \left[\frac{\eta(0)z^{n-1} + \eta(1)z^{n-2} + \cdots + \eta(n-1)}{z^{n-1}}\right]\bar{r}$$

Finally, $G_d(z)$ is obtained by dividing $\bar{U}(z)$ by $E(z)$.

3.1 Special Case: $n = 2$

The second-order case deserves special attention. When $n = 2$,

$$\begin{aligned}
G_d(z) &= \frac{v(0)z + v(1) + v(2)/(z-1)}{\eta(0)z + \eta(1)} \\
&= \frac{v(0)}{1 - dv(0)} \frac{z^2 + \frac{v(1)-v(0)}{v(0)}z + \frac{v(2)-v(1)}{v(0)}}{(z-1)(z+\alpha)}
\end{aligned}$$

where

$$\alpha \stackrel{\triangle}{=} \frac{\eta(1)}{\eta(0)} = \frac{1 - c'\Gamma v(0) - dv(1)}{1 - dv(0)}$$

Comparing with a *PI-Lead* controller

$$G_d(z) = K_P + K_I T_s \frac{z}{z-1} + \frac{K_D}{T_s}\frac{z-1}{z+\alpha} \qquad (16)$$

one can solve for the three gains $K_P,\ K_I,\ K_D$ and obtain

$$\begin{aligned}
K_P &= \frac{v(0)}{1 - dv(0)} - T_s K_I - K_D \\
K_I &= \frac{1}{T_s(1-\alpha)}\frac{v(2)}{\eta(0)} \\
K_D &= \frac{1}{(1 - dv(0))(\alpha - 1)}\{\alpha v(0) + v(1) + v(2)\}
\end{aligned}$$

Note that if $|\alpha| << 1$, then $G_d(z)$ can be reduced to a standard *PID* controller. Further research will reveal if this second-order controller may be used as a starting point in the design (tuning) of a PID controller for a higher order model of the process.

4 Examples

4.1 Example 1: A Second Order System.

Consider a second order continuous-time system with parameters

$$\{A,\ b,\ c,\ d\} = \left\{ \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix};\ \begin{bmatrix} 0 \\ 1 \end{bmatrix};\ [3\ 0];\ 1 \right\}$$

and zero output initial conditions. The sampling time is $T_s = 1$ and a unit step reference is applied. The compensator is

$$G_d(z) = \frac{-3.8891z^2 + 11.0979z - 1.4307}{z^2 - 6.0968z + 5.0968}$$

or in terms of the *PI-Lead* parameters (16),

$$G_d(z) = 0.2558 - 1.4104\frac{z}{z-1} - 2.7344\frac{z-1}{z+5.0968}$$

The closed-loop transfer function is found to be

$$\frac{Y(z)}{R(z)} = \frac{(z-2.7183)(z-0.1353)(z^2+1.5676z+1.3106)}{z^2(z-2.7183)(z-0.1353)}$$

where we see that the function of the compensator is to make the closed-loop characteristic equation equal to z^2. Figure 1 shows the error and the control sequences achieving their final values in two time steps.

4.2 Example 2: A Third Order System

The system is taken from Example 10-16 in [1]. The third-order plant is given by

$$G_p(s) = \frac{0.006702(s-3.622)(s-5.866)}{(s+1.05)(s+0.2231)(s+0.1054)}$$

and with $T_s = 1$ the zoh-discretization is

$$G_p(z) = \frac{0.05(z+0.5)}{(z-0.9)(z-0.8)(z-0.35)}$$

The minimum-time (deadbeat) controller derived in [1] is

$$G_{d1}(z) = \frac{20(z-0.9)(z-0.8)(z-0.35)}{(z+0.5)(z+1)(z-1)}$$

resulting in a response that follows a step reference after only *two* sampling periods. This compensator cancels all the poles and zeros of the process. On the other hand, the controller synthesized using Theorem 1 is

$$G_d(z) = \frac{13.3333(z-0.9)(z-0.8)(z-0.35)}{(z-1)(z+0.5-0.2887\jmath)(z+0.5+0.2887\jmath)}$$

which is seen to cancel only the process poles. The result is a deadbeat response that follows the step reference after *three* sampling periods. These responses are shown in Figure 2. Note that considerably more control activity is required of the minimum-time controller.

4.3 Example 3: A Flexible Structure.

A 2-mode, flexible arm system model was experimentally verified [16] to have the transfer function $G_p(s) =$

$$\frac{2.0625(s^2 + 0.3282s + 197.6)(s^2 + 1.935s + 2342)}{(s^2 + 0.113s + 56.85)(s^2 + 1.91s + 2280)}$$

Its zoh-discretization with $T_s = 1$ is given by $G_p(z) =$

$$\frac{2.0625(z^2 + 0.644z + 0.1554)(z^2 + 1.243z + 2.367)}{(z^2 + 0.6268z + 0.1481)(z^2 - 0.5843z + 0.8932)}$$

and the resulting deadbeat compensator is $G_d(z) =$

$$\frac{0.066465(z^2 + 0.6268z + 0.1481)(z^2 - 0.5843z + 0.8932)}{(z - 1)(z + 0.3371)(z^2 + 0.4043z + 0.1496)}$$

where once again the partial pole-zero cancellation function of the controller is evident. Figure 3 shows the error and control responses.

5 Conclusions

In this article, we have considered the problem of synthesizing the *coefficients* of a pole-zero compensator for SISO processes with the goal of achieving deadbeat characteristics. The design equations are derived using state-space notions, are simple to implement, and do not require an observer. When the process is of second-order, then the compensator is *PI-Lead*. Three examples were used to illustrate the results where, in particular, the performance of the n-step deadbeat controller is compared to that of a minimum-time (also deadbeat) controller. Further lines of research include using duality to describe the design of a deadbeat observer; handling of a process time delay; tracking of ramp and parabolic reference signals; analysis of inter-sample behavior since a deadbeat response in a discrete-time system does not guarantee a ripple-free response in a sampled-data system; avoidance of unstable process pole cancellation by compensator zeros; handling of process poles at the origin (continuous time) with multiplicity higher than one; and providing tuning and stability results for the two-step deadbeat compensator designed using the results in this paper and applied to a higher-order model of the system.

Acknowledgements: Research was partially funded by NASA/Stennis NAG13-03006 and by the University of Houston GEAR program I087518.

References

[1] Benjamin C. Kuo, *Digital Control Systems*, Saunders College Publishing, Second Edition, 1992.

[2] P.J. Kennedy, *Tracking enhancement with deadbeat feedforward compensation*, Proceedings of the IEEE Conference on Control Applications, vol.2, pages 1108 - 1115, Sept. 1992.

[3] L. Jetto, *Deadbeat controllers with ripple-free requirement for SISO discrete systems*, IEE Proceedings D Control Theory and Applications,V.137, Issue 5, Sept. 1990, pages 323 - 328.

[4] A. Emami-Naeini, *Deadbeat control of linear multivariable generalized state-space systems*, IEEE Transactions on Automatic Control, V.37, No.5, May 1992, pp.648 - 652.

[5] V. Eldem, H. Selbuz, *On the general solution of the state deadbeat control problem*, IEEE Transactions on Automatic Control, V.39, No.5, May 1994. pp.1002 - 1006.

[6] K. Kresselmeier, *Stabilization of linear systems in the presence of output measurement saturation*, Systems and Control Letters, V.29, pp. 27-30, 1996.

[7] M. Yamada, Y. Funahashi, and Z. Riadh, *Design of deadbeat control system considering worst case steady-state tracking performance*, Proceedings of the 36th IEEE Conference on Decision and Control, V.1, pp.351 - 356 Dec. 1997.

[8] Nam-Kiu Tsing, J. Lam, and H.K Tso, *Robust deadbeat regulation*, International Journal of Control, V.67, pp.587-602 1997.

[9] A. Casavola, E. Mosca, and P. Zecca, *Robust ripple-free deadbeat control design*, International Journal of Control, V. 72, pp. 564-573, 1999.

[10] Wataru Kase, Suboptimal Dead Multivariable Beat Controller for a Discrete-Time Plant with Measurement Noise, *Proceedings of the American Control Conference*, San Diego, California, June 1999, pp.810-814

[11] S. Tanaka, K. Furuta, *Generalized design of mixed H-infinity/deadbeat suboptimal controllers for SISO continuous-time servo systems*, Proceedings of the 39th IEEE Conference on Decision and Control, V.2, pp.12-15, 2000.

[12] T. Mita and Taek Kun Nam, *Control of underactuated manipulators using variable period deadbeat control*, Proceedings of the IEEE International Conference on Robotics and Automation, V.3, pp.2735 - 2740, 2001.

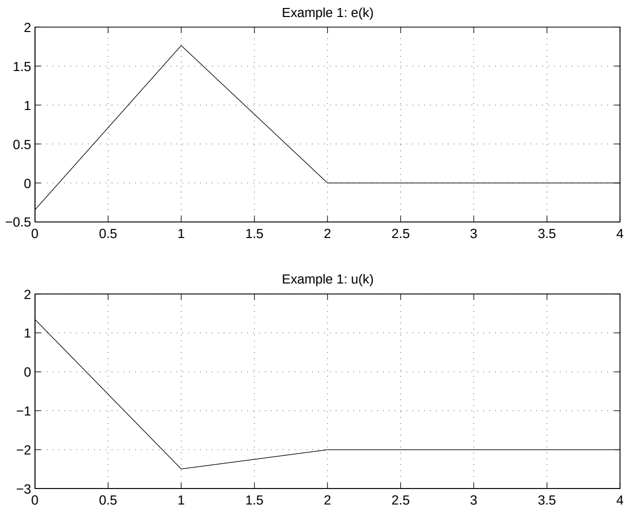

Figure 1. Example 1: Error and Control Sequences.

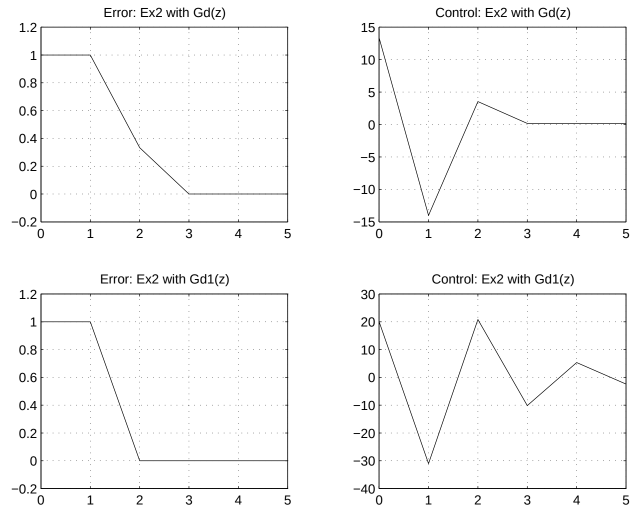

Figure 2. Example 2: Error and Control Sequences with $G_d(z)$ and $G_{d1}(z)$.

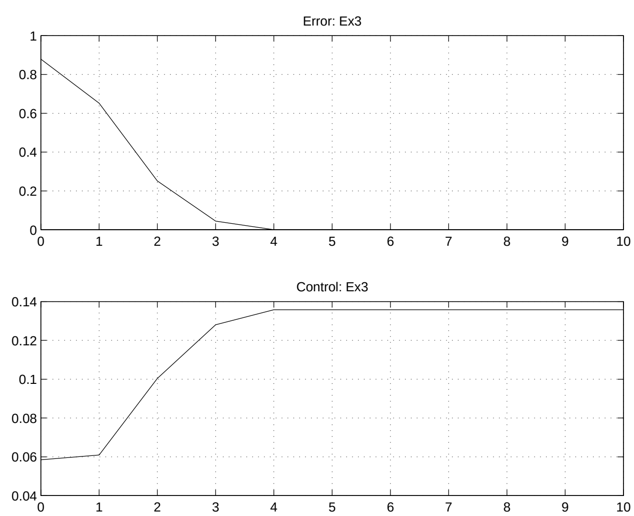

Figure 3. Example 3: Error and Control Sequences.

[13] F. Blanchini and F. A. Pellegrino, *Relatively optimal control and its linear implementation*, IEEE Transactions on Automatic Control, V.48, No.12, Dec. 2003, pp.2151 - 2162.

[14] L. H. Keel, J. I. Rego, and S. P. Bhattacharyya, *A new approach to digital PID controller design*, IEEE Transactions on Automatic Control, V.48, No.4, April 2003, pp.687 - 692.

[15] James Lam, H. K. Tso, and N. K. Tsing, *Robust Deadbeat Regulation*, International Journal of Control, Vol.67, No.4, pp. 587-602, 1997.

[16] P. T. Kotnik, S. Yurkovich, and Ü. Özgüner, *Acceleration Feedback for Control of a Flexible Manipulator Arm*, Journal of Robotic Systems, Vol.5, No.3, pp. 181-196, 1988.